PACKET COMBINING OVER RAYLEIGH CHANNELS USING SIGNAL-TO-NOISE RATIO INFORMATION AND DETECTION BY THE MAXIMUM A-POSTERIORI CRITERION

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ABSTRACT

We present and analyze a packet combining strategy for wireless networks with slow Rayleigh fading. The scheme is based on adding the current signal-to-noise ratio (SNR) as an overhead to the packet and packet combining using the maximum a posteriori (MAP) criterion. We consider single and multiple wireless hops, and we perform comparisons against the optimum case, the maximum ratio combiner (MRC). For the single wireless hop, we show that the error probability curve is very close to the optimum. In addition, we study the effect of selection and an alternative based on averaging over the channel statistics that has poorer error performance, but needs less processing and overhead. In multiple wireless hops, some nodes act as relays and the error probability increases. However, combining several branches, each with two hops, allows for diversity order equal to the number of branches. We demonstrate the performance of the proposed strategy by computer simulations. The fusing packet scheme that is presented in this paper is adequate for sensor networks.

1. INTRODUCTION

In the literature, there is previous work on packet combining in single user and multiuser systems. In [1], coding is introduced to combine packets, where overhead bits can be added to the packet to indicate its reliability. In [2] the performance of a DS/CDMA ALOHA with packet combining is presented. There, diversity is achieved with several received copies of one packet. Similar combining for controlling retransmission is presented in [3] and packet combining using the output of a multiuser minimum mean-square error detector is described in [4].

The systems referred so far perform the process of post-demodulation and combining at the same site. However, it is not always possible, or it is very expensive to have available post-demodulation information of every bit at a central point so that one can perform combining. In the literature, for such systems several solutions have been proposed. In [5], a coded source packet is routed through multiple routes and at the destination before decoding, the demodulated packets are combined at bit level. In ad hoc networks studied in [6], the SNR at a receiver of a multi-hop is used to define a new variable called erristor. This variable simplifies the design of time diversity and selection combining schemes in simple systems.

Cooperative diversity is a field that has had high activity recently. Many papers have appeared since the term was coined in [7]. In [8] it is presented an error-correction scheme for sensor networks. The relay channel appears as part of cooperative systems [9]. In [10], it is proven that decode and forward transmission has a diversity order of one and thus is the diversity order that is expected in diversity branches with two wireless hops. The system that we propose is not cooperative but uses decode and forward for nodes acting as relays.

As overhead for combining at a node, we propose to use the SNR of the received packet. This information and the bits of the packet are the input to an optimum MAP combiner. This system has the advantage of simplicity over the systems mentioned above. It does not need coordination between nodes and terminals and can be implemented with minor changes in existing wireless systems. Comparing our system with cooperative systems with links with multiple hops, we see that the latter can obtain diversity orders better than one with specific relaying at the expense of more complexity [10]. On the other hand, the simulations on combining single hop branches show that the error probability figures are close to the ones of MRC [11].

2. SYSTEM MODEL

We consider a wireless system where terminals reach several nodes. They send packets of information and we consider that each packet is an independent entity, as it is the case in the Internet Protocol. From the nodes the packets arrive to one or more processors where a decision is taken about their final destination. At each processor we can have several copies

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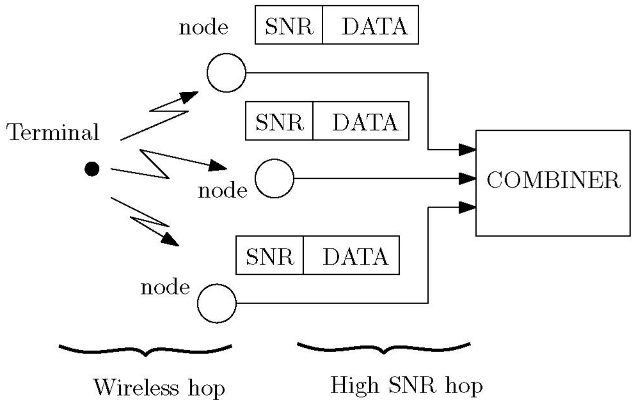

Fig. 1. Single wireless hop - Several branches

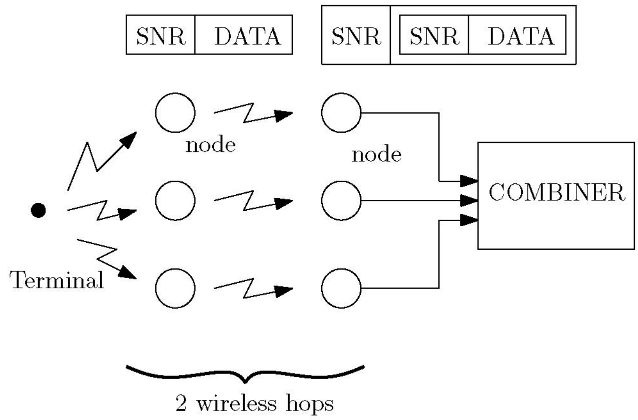

Fig. 2. Two wireless hops - Several branches

of the same packet and we want to combine them. We consider two sections in the packet trip: a wireless section where a packet can suffer from channel impairments and a clean section, cable or wireless, where the impairments are minor and are not considered. This situation is frequent in practice, where stations are connected to some backbone system.

In the wireless section, there is always a hop between the terminals and the nodes, but the nodes can resend packets to other nodes before the packets reach the clean section. We consider two extreme cases: case (a) depicted in Fig. 1 where the terminal reaches several nodes and these are linked to the processor, in this case the combiner, and case (b), Fig. 2, a single branch is formed by two nodes, one of them is the relay and the branches are connected to the combiner. We can also consider situations with branches with one hop and other branches with two hops. For simplicity we limit the number of hops to two. The results we obtain can be extended easily to more hops.

Whenever terminals and/or nodes move, channels change, which is a frequent scenario in wireless systems. We consider that the channels' changes are much slower than the reference periods of signals. Therefore, the channels are modeled by Rayleigh attenuation [11] affecting likewise all the bits of one packet. We also consider that the system has no inter-symbol interference (ISI) and multi-user interference (MUI) and that the used modulation is BPSK.

3. SINGLE WIRELESS HOP

A terminal transmits a bit in the $i-$th bit period, and K different nodes detect the information. The real parts of their adapted filter outputs, i.e., the sufficient statistics, are

$$y_i^{(k)} = \alpha_k^{(k)} b_i + n_i^{(k)} \quad k = 1, 2, \cdots, K \tag{1}$$

where $b_i \in \{+1, -1\}$ is the transmitted bit, $\alpha^{(k)}$ is Rayleigh distributed attenuation suffered by the signal, and $n_i^{(k)} \sim N(0, \sigma/\sqrt{2})$ is the noise. The instantaneous SNR associated with a particular bit at a node k is defined as $\gamma^{(k)} = {\alpha^{(k)}}^2/{\sigma^{(k)}}^2$. This value is representative for the whole packet. Observe that we have assumed that all the nodes have the same noise parameters.

The detector of a node quantizes $y_i^{(k)}$ with two levels by taking its sign and obtaining the bit corresponding to the $i-$th bit period. The same procedure is perfomed for the remaining bits of the packet. The value of the SNR $\gamma^{(k)}$ is quantized, coded for error protection, and added to the packet. This information is used by the combiner.

3.1. MAP combining

Let b_i be the transmitted bit and $s_i^{(k)}, k = 1, 2, \cdots, K$, the received signs by the K different detectors. Given $\gamma^{(k)}$ the MAP detector decides in favor of $b_i = 1$ if

$$\begin{aligned} &\sum_{k=1}^{K} \log\left(P(s_i^{(k)}|b_i = 1, \gamma^{(k)}\right) \\ &\quad > \sum_{k=1}^{K} \log\left(P(s_i^{(k)}|b_i = 0, \gamma^{(k)}\right) \end{aligned} \tag{2}$$

where

$$\begin{aligned} P(s_i^{(k)} = 1|b_i = 1, \gamma^{(k)}) &= Q(\sqrt{2\gamma^{(k)}}) \\ P(s_i^{(k)} = -1|b_i = -1, \gamma^{(k)}) &= Q(\sqrt{2\gamma^{(k)}}) \\ P(s_i^{(k)} = 1|b_i = -1, \gamma^{(k)}) &= 1 - Q(\sqrt{2\gamma^{(k)}}) \\ P(s_i^{(k)} = -1|b_i = 1, \gamma^{(k)}) &= 1 - Q(\sqrt{2\gamma^{(k)}}) \end{aligned} \tag{3}$$

where $Q(\cdot)$ is the complementary of the cumulative distribution function of the standard Gaussian distribution.

The SNR MAP combiner (SMC) uses equation (2) and the instantaneous SNR $\gamma^{(k)}$ that must be estimated by the detectors and prepended to the packets for combining. Although, we did not obtain closed expressions for the probability of error, we conducted efficient simulations which are reported in the Section on Numerical Results. There we show that the probability of error curve of this detector is close to the one that applies MRC.

Several variants that introduce simplifications of the SMC combiner are explained below.

3.1.1. *Selection Combining (SC) with instantaneous SNR*

In [12], optimum selection combining by the MAP criterion was derived. For each bit the log-likelihood ratio (LLR) of each branch has to be sent. Our scheme can be used with selection diversity and a closed expression for the error probability can be obtained. The advantage of our method is that it does not send the LLR information of each bit, but instead it uses the SNR of the whole packet.

Choosing the bit of the detector with best LLR implies that a decision is based on the bit of the detector with the highest SNR, which can easily be shown. The probability of error is

$$P_e = K \times P_r\left(l^{(i)} \leq 0, \gamma^{(i)} = \max_{k=1,2,\cdots,K}\{\gamma^{(k)}\}\right) \quad (4)$$

where $l(i)$ is the LLR of the $i-$th detector.

The probability of error can be evaluated using the density of $\gamma^{(k)}$ and integrating it over the region where $\gamma^{(k)} \geq 0, k = 1, 2, \cdots, K$. The resulting probability of error is

$$P_e = \frac{1}{2}\sum_{k=0}^{K}\binom{K}{k}(-1)^k\frac{1}{\sqrt{\frac{k}{\bar{\gamma}}+1}} \quad (5)$$

where $\bar{\gamma}$ is the average SNR.

3.1.2. *Average MAP combining*

We can avoid to estimate and transmit the instantaneous SNR by averaging. In this case there is no need to send an overhead with the SNR's. The detectors inform of the average $\bar{\gamma}^{(k)}$ to the combiner only once.

The average MAP makes the decisions in favor of $b_i = 1$ according to

$$\begin{aligned}&\int_0^\infty \sum_{k=1}^{K}\log\left(P(s_i^{(k)}|b_i = 1, \gamma^{(k)}\right) d\gamma^{(k)} \\ &> \int_0^\infty \sum_{k=1}^{K}\log\left(P(s_i^{(k)}|b_i = 0, \gamma^{(k)}\right) d\gamma^{(k)}.\end{aligned} \quad (6)$$

The detector can be implemented by calculating the integrals of the form

$$I(\bar{\gamma}) = \int_0^\infty \log\left(Q(\sqrt{2x})\frac{x}{\bar{\gamma}}e^{-\frac{x^2}{2\bar{\gamma}}}dx\right. \quad (7)$$

which appear in (6) and arranging them in a table.

In order to study the performance of this detector and compare it with known results from the literature, we consider that all detectors have the same average SNR $\bar{\gamma}$. For this particular case, the detector (6) results in a majority criterion. In other words, if the number of +1's received from the K detectors is larger than the number of -1's, the combiner decides a +1 and vice versa.

Each of the K detectors are independent and if we denote with $P_s = P_s(\bar{\gamma})$ the probability of error over a slow Rayleigh fading channel [11], then the error probability of the combiner is

$$P_e = \sum_{k=\lceil K/2\rceil}^{K}\binom{K}{k}P_s^k(1-P_s)^{K-k} \quad (8)$$

This probability of error can be decreased a little when K is an even number and adopting a random test for $K/2$ errors.

Average MAP combining has a reduction of the diversity order. From equation (8) it is easy to see that the diversity order [13] is reduced to $\lceil K/2\rceil$.

4. MULTIPLE WIRELESS HOPS

The MAP combining criterion can be extended to the multi-hop case. For simplicity we only consider two hops. As in the previous case, the nodes prepend information about the instantaneous SNR to the output of the matched filter $y_i^{(k)}$, which has two quantized levels. This information is protected by a good channel code to avoid perturbation in the combining process. Here it is assumed that the SNR information is recovered without errors.

Let b_i be the transmitted bit and $s_i^{(k)}, k = 1, 2, \cdots, K$ be the received signs by the K different detectors of the second set of nodes. We denote the SNRs after the first and second hops by $\gamma_1^{(k)}$ and $\gamma_2^{(k)}$, respectively, and by $s_{1,i}^{(k)}$ a received sign by the $k-$the detector from the first set of nodes. The MAP detector decides in favor of $b_i = 1$ if

$$\begin{aligned}&\sum_{k=1}^{K}\log\Big(\sum_{s_{1,i}^{(k)}}P(s_i^{(k)}|b_i = 1, \gamma_2^{(k)}, s_{1,i}^{(k)}) \\ &\qquad \times P(s_{1,i}^{(k)}|b_i = 1, \gamma_1^{(k)})\Big) \\ &> \sum_{k=1}^{K}\log\Big(\sum_{s_{1,i}^{(k)}}P(s_i^{(k)}|b_i = 0, \gamma_2^{(k)}, s_{1,i}^{(k)}) \\ &\qquad \times P(s_{1,i}^{(k)}|b_i = 0, \gamma_1^{(k)})\Big)\end{aligned} \quad (9)$$

where the involved probabilities are evaluated in a similar way as in (3) and are omitted here.

Note that we can combine branches with a single hop and others with two hops using the appropriate sums in equations (2) and (9). It is difficult to obtain a closed expressions for the probability of error. In the next section, we present and

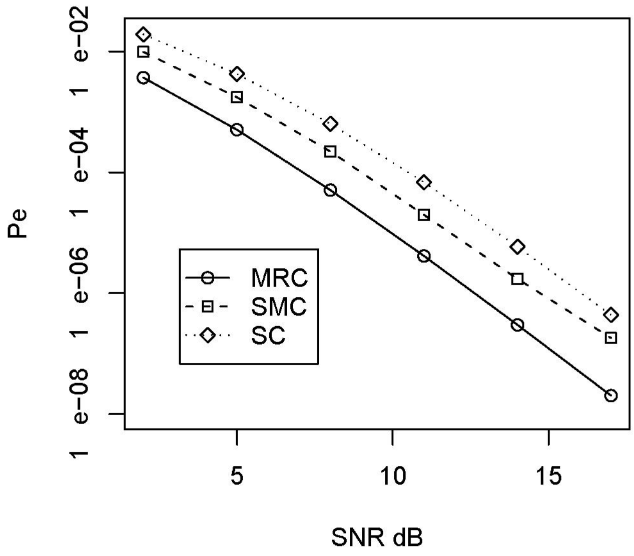

Fig. 3. Probability of error with instantaneous SNR.

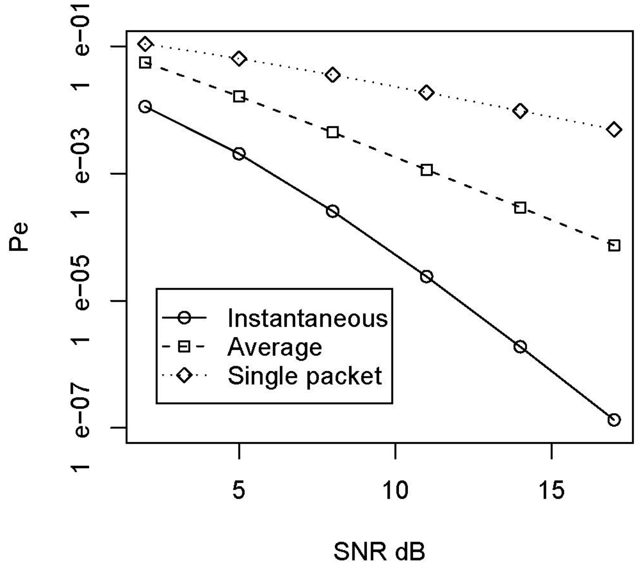

Fig. 4. Probability of error with average attenuation.

analyze results that are obtained by using Monte Carlo simulations.

5. NUMERICAL RESULTS

We compared the figures of probability of error obtained by our system against the figures of the optimum MRC, where the LLR of each bit has to be sent [11]. We considered that the average SNRs at the input of all detectors are the same, and we combined four detectors, which is a typical example in the literature. In Fig. 3 we present the results obtained with SMC.

The results show that the loss for sending the instantaneous SNR of the whole packet is about 2 dB compared with the MRC. If, for instance, we have a system with a packet length of 64 bits, we use 8 bits for coding the LLR of each bit, and with MRC we would need $64 \times 8 = 512$ bits of LLR information for the whole packet. With SMC, we code the instantaneous SNR with 8 bits and we need 64 bits for the signs. However, we should increase the transmitted power by two dB to maintain the error probability of the MRC.

In the same Fig. 3 we presented the results of SC as given by (5). The SC curve is close to the curves of the SMC and MRC [12]. There is not a practical advantage on using selection combining as adding two or more quantities is as fast as choosing one of them.

In Fig. 4 we show the probability of error of equation (8) when four nodes with equal SNRs are combined with average MAP. For comparison, we presented the probability of error obtained with instantaneous attenuation and with a single detector with Rayleigh fading. Even though the error figures are worse than the ones with average combining, the amount of information to be sent is much smaller because it is necessary to estimate and inform only once about the mean value of the SNR. In terms of dBs, the gain over a single detector is about 10 dB.

Two successive hops lead to an increase of the probability of error, as can be seen in Fig. 5. There we show the curves of probability of error versus average SNR at the receivers of the nodes for links with a single hop and with two hops, both in Rayleigh fading. In the link with two hops, the average SNR of both receivers is assumed equal. The results of combining four branches, each with two hops, are shown in Fig. 6. In the same figure, the MRC and one hop SMC (1H-SMC) probabilities of errors are superimposed for comparison. The three curves have the same slope and thus their diversity order is the same. The attenuation caused by the two hops is about three dB, compared with the 1H-SMC. The combining presented in Fig. 6 is an extreme case as all the branches have two hops. In a more real case we expect that branches with one hop and others with two hops are present at the same time and thus, the curve that shows the probability of error should appear between the curves of 1H-SMC and 2H-SMC.

6. CONCLUSIONS

We have presented and analyzed a packet combining strategy for wireless networks with slow Rayleigh fading. We proposed a scheme where the detected bit and the instantaneous SNR common to all bits in the packet is all the information needed to combine the packets. The overhead information per packet is small, i.e., it is only the SNR. The simulation results show small degradation in performance in comparison to the optimum MRC case. We have considered the case of using average instead of instantaneous SNRs. Even though the degradation of the probability of error compared to that of

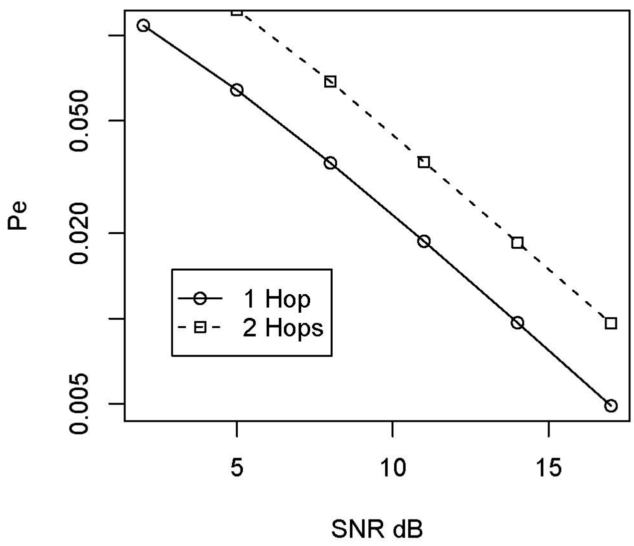

Fig. 5. Probability of error for a branch with two hops.

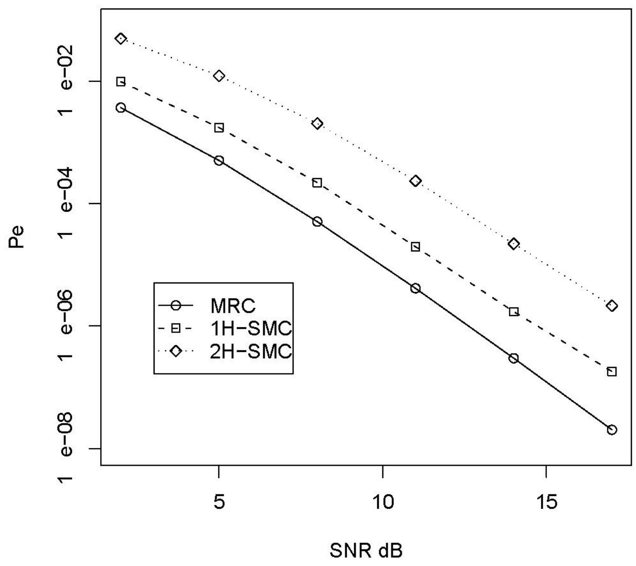

Fig. 6. Probability of error when combining four branches with two hops.

using instantaneous case is significant, it avoids a continuous estimation of the SNRs. This information is sent only after long periods of time. We also studied the case of links with two hops forming a branch. We showed through simulations that multihops degrade the probability of error. However, by combining several branches using a MAP criterion yields a diversity order equal to the number of the branches. We can mix system branches with one hop with those of multihops.

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